Worst-case to Expander-Case Reductions: Derandomized and Generalized

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WEIZMANN INSTITUTE OF SCIENCE

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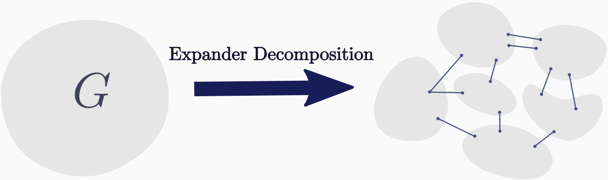
Whenever $\phi = \Omega(1)$, we simply say that G is an expander.

Expander Decompositions



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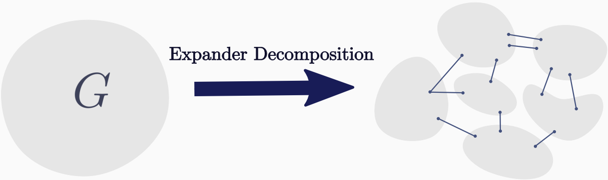

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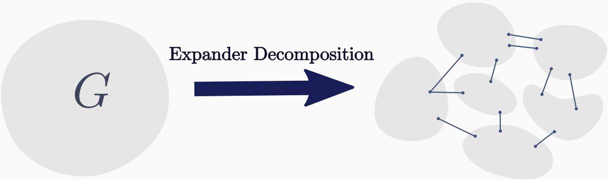

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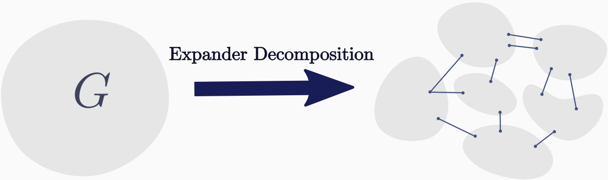

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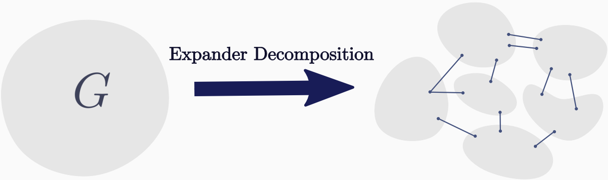

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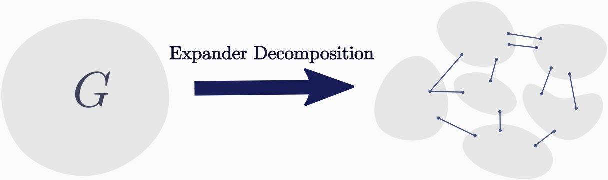
Note that the decomposition is non-trivial only for $\phi = \frac{1}{poly\log(n)}$ because of the second property.

A Self-Reduction: The Expander Decomposition Method

One can self-reduce a problem to expanders using the expander decomposition method.



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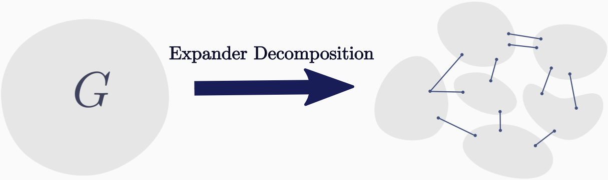

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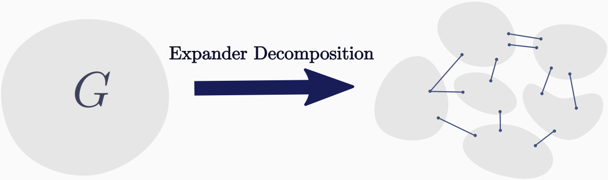

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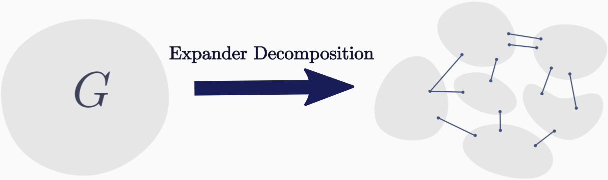

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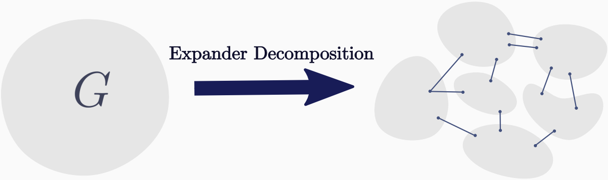

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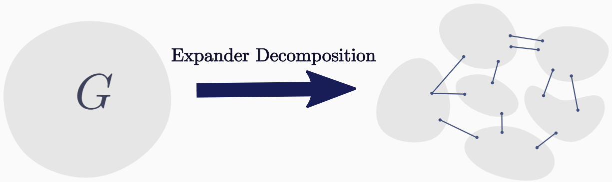

The expander decomposition method has been key to many breakthroughs in recent years:

- Max-flow in $m^{1+o(1)}$ time.
- *Dynamic* minimum spanning tree in $n^{o(1)}$ update time.
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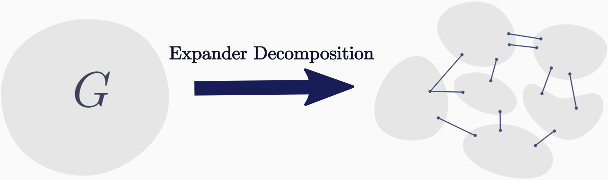

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Question 2

Is expander decomposition the key to solving my problem?

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If problem $\mathcal{A}$ admits a Direct Reduction, then:

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If problem $\mathcal{A}$ admits a Direct Reduction, then:

- Hardness - $\Omega(1)$-expanders are worst-case instances of $\mathcal{A}$.
- Conceptual message - The expander decomposition method is useless against $\mathcal{A}$.

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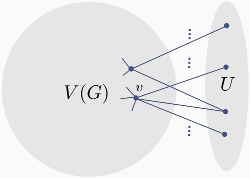
[Abboud and Wallheimer, 2023]
The following problems admit direct reductions:

- k-Clique Detection
- Maximum Matching
- Vertex Cover
- Minimum Dominating Set

The Randomized Core Gadget of AW'23



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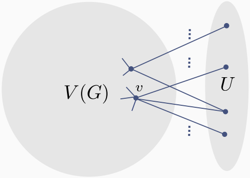

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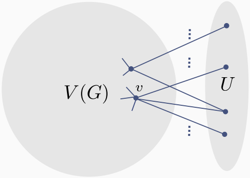

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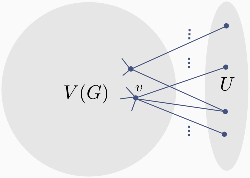

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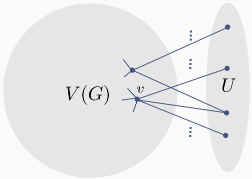

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High level idea: Random walks mix well in this graph.

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Question 3

Are expanders worst-case instances in the fully-dynamic setting?

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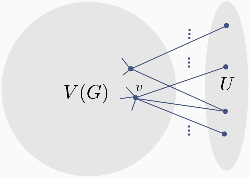

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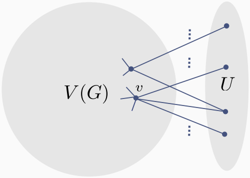

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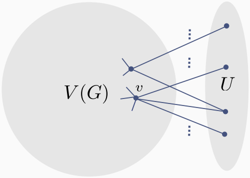

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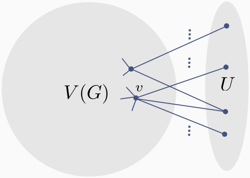

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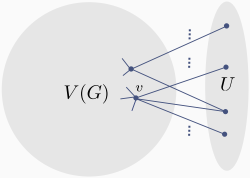

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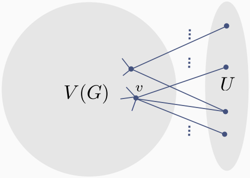

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- $d = \Theta(n)$. The blowup is too big.
- $d = \Theta(m/n)$. Same problem as before. E.g., a graph with $m = O(n)$ edges that contains an isolated $\Theta(\sqrt{n})$-clique.

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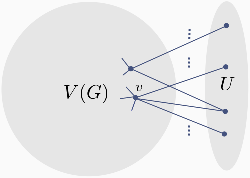

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Open Question

Is there an explicit construction of expanders that, given a degree sequence $(\deg(v_1), \deg(v_2), \ldots, \deg(v_n))$, construct an $\Omega(1)$-expander with this degree sequence?

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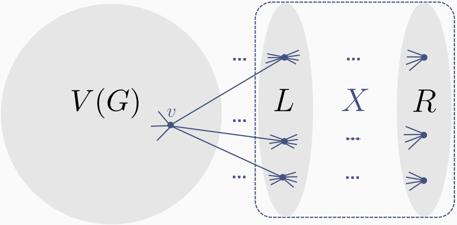

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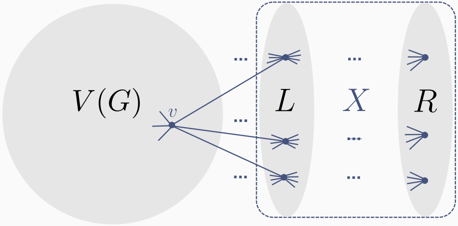

1. For every $v \in V$, add $\deg_G(v)$ neighbors in L using a deterministic load-balancing rule.

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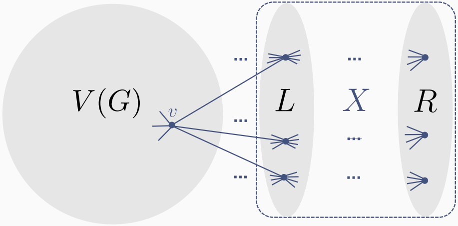

1. For every $v \in V$, add $\deg_G(v)$ neighbors in L using a deterministic load-balancing rule. Now L is almost d-regular, where $d = 2m/n$.

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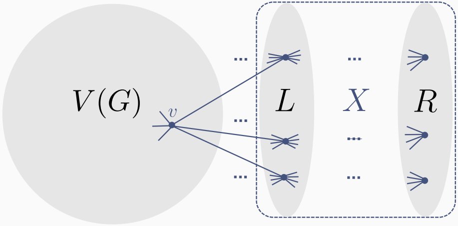

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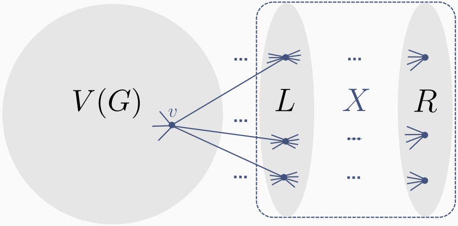

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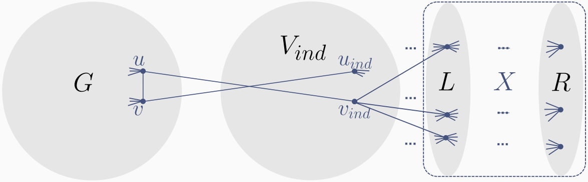
Claim: G_{exp} is an $\Omega(1)$-expander, and its size is $O(m + n)$.

Example: Direct Reduction for k-Clique Detection



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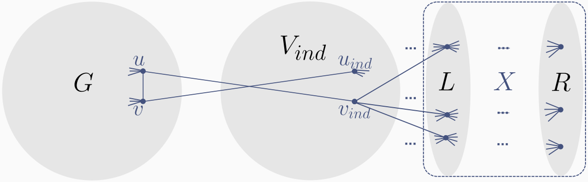

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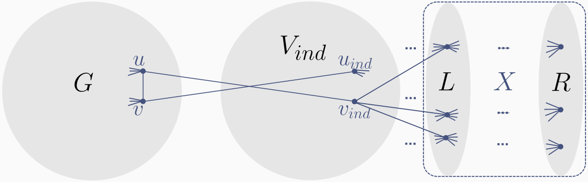

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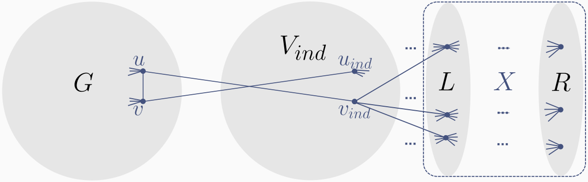

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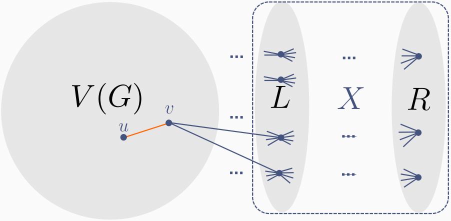
Claim: G_{exp} is an $\Omega(1)$-expander and the number of k-cliques in G_{exp} is $k\times$(the number of k-cliques G).

Attempt to Dynamize the Core Gadget



- For every insertion of an edge uv in G, add two edges from u, v to L.

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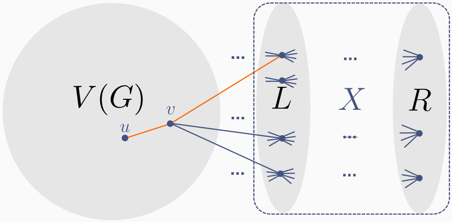

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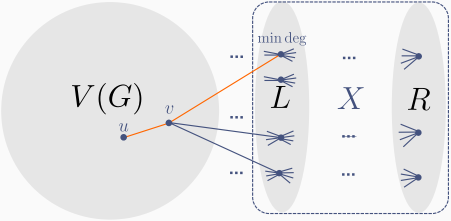

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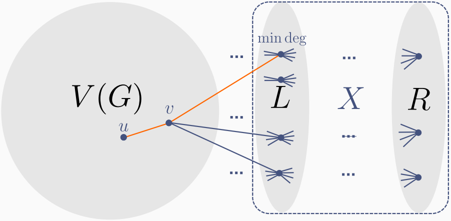

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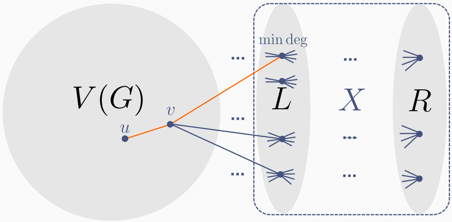
Subtle issue - we may introduce parallel edges to the graph.

Fully-Dynamic Core Gadget



Solution: (greedy + lazy) strategy.

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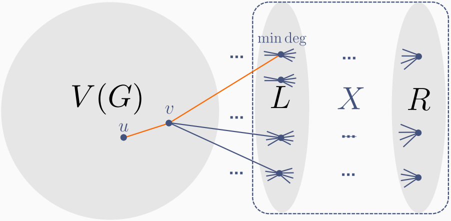

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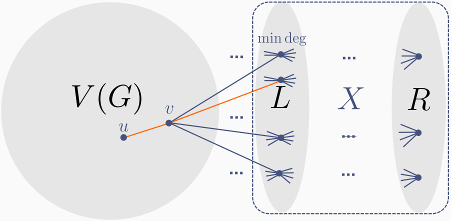

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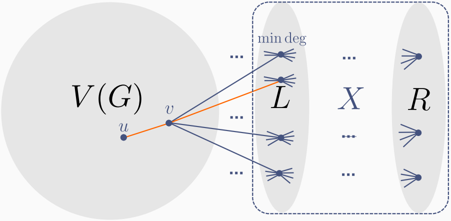

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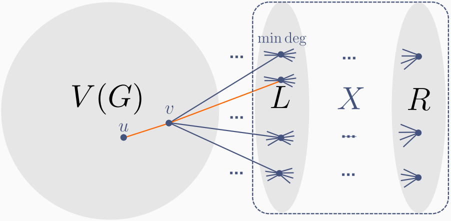

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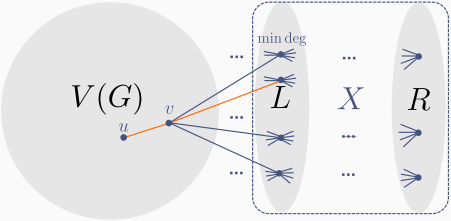

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Claim: The degrees in L are in $\Theta(2m/n)$ for at least $\Omega(m+n)$ updates.

Summary and Open Questions

Our Results

The following problems admit deterministic, fully-dynamic, worst-case to expander-case reductions:
k-Clique Detection, Maximum Matching, Dynamic Densest Subgraph, Max-Cut, Minimum Vertex Cover, Minimum Dominating Set.

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Open Question

- Demanding that the reduction outputs a ϕ-expander, where $\phi = \Omega(1)$, might be too coarse. Some problems (e.g., Triangle Detection) remain hard on $1/100$-expanders but become easy on $(1/2 - o(1))$-expanders. Find the exact threshold.
- Algorithms parameterized by ϕ?